Propositions accompanying the PhD thesis

The arithmetic of maximal curves, the Hesse pencil and the Mestre curve

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1. Let C be a curve of genus g over $\mathbb{F}_q$ with non-supersingular Jacobian variety. The upper bounds presented in Chapter 1 of the thesis for the case $g = 1$ on the degree of $k/\mathbb{F}_q$ such that C is maximal over k generalize to the case $g > 1$.

2. For infinitely many primes p there exists a genus 2 curve C over $\mathbb{F}_p$ such that C is maximal over $\mathbb{F}_{p^3}$.

3. The hyperelliptic curve C over $\mathbb{F}_{17}$ defined as $y^2 = x^9 + 7x^5 - x$ is maximal over $\mathbb{F}_{17^3}$ and the twist of C defined as $y^2 = x^9 + 11x^5 + x$ is minimal over $\mathbb{F}_{17^3}$.

4. The work of Katz on Frobenius trace ratios in the Legendre family of elliptic curves over finite fields does not carry over in a natural way to the Hesse pencil.

5. There are at most countably many $j \in \mathbb{C}$ such that the Jacobian variety of the curve $D_{a,b}$ is isogeneous to a product of elliptic curves, where $a, b \in \mathbb{C}$ satisfy $j = 1728\frac{4a^3}{4a^3+27b^2}$ and $D_{a,b}$ is defined as
$$y^2 = (x^3 + ax + b)(ax + b)(ax - 3b).$$

The following two propositions are related to Chapter 6 of the thesis:

6. The maximal exponent 4 extension of $\mathbb{Q}$ unramified outside 2, 5 and ∞ has degree 2^{14} over $\mathbb{Q}$ and is the splitting field of
$$(x^8 + 4x^6 + 4x^4 - 2) \cdot (x^8 + 4x^6 + 4x^4 - 5) \cdot (x^8 + 4x^6 + 4x^4 - 10) \cdot (x^{16} + 6x^{12} - 4x^{10} + 8x^8 + 8x^6 - 4x^4 - 8x^2 + 4).$$

7. The number of isogeny classes of abelian surfaces A over $\mathbb{Q}$ with good reduction at every prime $p \neq 2, 5$ and $\mathbb{Q}(A[2])/\mathbb{Q}$ a 2-extension is at most $2.1 \cdot 10^{1435}$.

8. The scientific community would benefit from a professionally maintained collection of errata of scientific publications.